Sovereign Debt, Currency Composition, and Financial Repression*

Teresa Balestrini
University of Minnesota

Leonardo Barreto
University of Minnesota

Carlos Bolivar
University of Minnesota

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Abstract

This paper examines the interaction between the currency denomination of sovereign debt and the composition of its holders. We document that, in emerging economies, local-currency bonds constitute the main instrument of government debt and are predominantly held by domestic investors. We develop a framework that characterizes the trade-offs governments face when domestic and foreign demand for bonds respond differently to policy changes. Domestic investors prefer local-currency bonds because these assets provide insurance against distortionary taxation. The government internalizes how currency denomination influences domestic demand and default risk, generating a novel endogenous link between the composition of bondholders and the choice of currency. Even abstracting from the standard hedging benefits against output fluctuations, it remains optimal for the government to issue local-currency debt to stimulate domestic demand, consistent with empirical evidence. Finally, we show that imposing minimum domestic holdings of foreign-currency bonds through financial repression can implement the optimal allocation without relying on local-currency issuance.

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1 Introduction

Sovereign debt markets exhibit four fundamental characteristics. First, governments typically issue debt denominated in both local currency (LC) and foreign currency (FC). Second, this debt is held by a mix of domestic and foreign investors. Third, LC bonds constitute the main debt instrument for governments in emerging markets, and fourth, they are mainly held by domestic investors.

In this paper, we study the relationship between the composition of bondholders and the currency denomination of sovereign debt, and analyze how this interaction shapes the optimal currency composition of sovereign bonds. In our analysis, we propose a new mechanism through which uncertainty about future taxes generates a bias among domestic investors in favor of LC bonds. We also show how this bias affects the optimal currency composition of sovereign bonds, as the government uses denomination strategically to affect who holds its bonds.

To develop these ideas, we first present a two-period model in which the government issues two types of bonds in a market populated with domestic and foreign investors. As in Lucas and Stokey (1983a), the government faces an exogenous expenditure that must be financed using distortionary taxes. Moreover, the government can use debt subject to default risk to smooth tax distortions. The key innovation of our model is that the government issues two types of debt—LC and FC—in an integrated market, so that the bondholder composition of both types of bonds emerges as an equilibrium outcome rather than as a direct policy choice. We characterize optimal portfolio choices for domestic and foreign investors. Next, we study the optimal currency composition of sovereign bonds in this environment. Finally, we present empirical evidence supporting the main mechanisms in the model.

Table 1 shows the decomposition of central government debt securities for 17 emerging markets. We categorize bonds by currency denomination (LC and FC) and investor type (domestic and foreign), yielding four different groups. On average, 74% of the bonds are denominated in LC. In addition, domestic investors constitute the largest group of bondholders in the sovereign debt market, holding an average of 68% of the debt. Interestingly, 91% of the bonds held by domestic investors are denominated in LC. The first result of the paper is to rationalize this currency bias by showing how uncertainty about future taxes generates incentives for domestic investors to hold more debt denominated in LC. In particular, we solve the optimal portfolio choice of domestic investors for any government polices of FC and LC debt and analytically show that they have a preference to hold LC debt.

Table 1: Taxonomy of Sovereign Debt

	Domestic lenders	**Foreign lenders**
Local currency debt	62 %	12 %
Foreign currency debt	6 %	20 %

Source: Arslanalp and Tsuda (2014)

In our model, domestic investors use government bonds as a hedge against fluctuations in taxes.

Because the government must raise taxes to repay debt, higher taxes are linked to a higher value of government bonds. In this way, government bonds act like insurance: they real value increase when tax levels are high. In addition, when the government increases the LC debt beyond the level that minimizes the volatility of taxes, two effects increase the domestic demand for these bonds. First, more LC debt increases the volatility in taxes, so domestic investors save more and end up holding a larger share of the debt. Second, since the value of the LC bond and taxes move together, domestic investors have higher incentives to buy LC bonds, as they provide better insurance compared to FC bonds.

A second important mechanism in the model is a disagreement between the government and domestic investors regarding their optimal demand for bonds. For the government, higher domestic demand for bonds serves as a commitment device that reduces default risk by lowering the future cost of repayment[1]. In addition, the government, acting as a monopolist, internalizes the effect of increased domestic demand on bond prices. In contrast, the representative domestic investor does not consider the impact of their demand on prices. As a result, the government has an incentive to manage the composition of the bondholders by increasing domestic demand, which, in turn, raises the prices of the bonds. Behind this result is an envelope condition that does not hold because domestic investors do not internalize the effect of their demand on prices. We derive a Generalized Euler Equation (GEE) that shows how the government internalizes the influence of currency composition on domestic demand for bonds.

Building on these results, we now turn to the study of optimal government policy. We extend the analysis of the optimal currency composition of debt in Perez (2018) in two critical dimensions. First, we consider a government that can default on its debt and study the interplay between the risk of default and the currency composition of the debt. Second, we explore the relationship between currency denomination and bondholder composition. In the model, the government faces a new trade-off when choosing the currency composition of its debt: balancing the benefits of providing insurance against fiscal risk with the objective of increasing domestic demand for debt.

The intuition behind this trade-off is as follows. Consider a scenario in which the government increases the share of debt denominated in LC beyond the level that minimizes fiscal risk. This increased share of LC debt has two opposing effects. On the one hand, it raises economic risk by exposing the fiscal budget and taxes to greater volatility. However, it increases the domestic demand for bonds, improving the commitment of the government to repay and reducing the risk of default. Focusing on the case where LC debt does not provide any insurance for the government, so that the level of LC debt that minimizes fiscal risk is zero, we provide an analytical proof showing that the government finds it optimal to issue a positive level of LC debt in order to increase the share of domestic debt.

To further inspect the mechanism of the model. we study the interaction of the optimal currency composition of debt and Financial Repression. Following Chari, Dovis, and Kehoe (2020), we extend the model and consider a government that can directly choose domestic demand for government bonds by using financial repression. We show that when financial repression has no cost, the government chooses the currency composition of debt to reduce volatility in the fiscal budget as in Ottonello and Perez (2019).

1. See Bolivar (2023), Mallucci (2022) and Perez (2018) for examples of models with similar mechanisms.

The theory has two key implications. First, when the government increases the LC debt, the domestic demand increases more than when the government issues debt denominated in FC. We validate this mechanism in the data by studying the empirical relation between the supply of sovereign bonds and the share held by domestic investors. Extending Broner et al. (2022a), we compare the correlation of higher government debt with the domestic demand for bonds, when the government increases the supply of LC or FC bonds. We find a stronger positive comovement between the supply of LC bonds and the share of domestic debt compared to FC bonds, supporting our theoretical mechanism. A second key implication of the model is that the government can influence the share of domestic debt by changing the currency composition of the debt. We validate the mechanisms by showing a positive correlation between the share of LC debt and the share of domestic debt.

1.1 Related Literature

This paper builds on the literature on currency and bondholder composition of sovereign debt, particularly in emerging markets. First, our paper is related to the quantitative literature on sovereign debt, following the work of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008). Most of this literature has focused on the fourth category of Table 1, that is, the debt in FC held by foreign lenders.

There are several recent contributions to the literature on the currency composition of debt in emerging economies following Eichengreen and Hausmann (1999). These models analyze governments' currency choices, focusing on the trade-offs between the hedging benefits of LC and the incentive benefits of FC debt (see the second column in Table 1). As emerging market governments gain credibility in their monetary policy, Ottonello and Perez (2019) and Engel and Park (2018) argue that they are increasingly borrowing in LC, overcoming the "original sin". Likewise, Du, Pflueger, and Schreger (2020) suggest that governments unable to commit to an inflation policy rule borrow more in FC. Lee (2021) highlights that despite the increased credibility in their monetary policy, emerging markets continue to borrow substantially in FC due to high exchange rate volatility. In an environment with a constant real exchange rate and no monetary policy, we argue that there is an additional incentive to issue in LC, as it increases domestic holdings of government bonds.

The composition of the bondholders plays a crucial role in determining the risk of sovereign default, as shown in the models by Sunder-Plassmann (2020) and D Erasmo and Mendoza (2021). Similarly to our framework, they consider an economy where the government is benevolent and cares about domestic investors, so the risk of default depends on the bondholder composition. These papers focus on the lower row of Table 1, where the government issues debt in foreign currency only, and the bond market is populated by domestic and foreign lenders. In contrast, in our paper, the bondholders' composition is an equilibrium outcome.

Within the literature on bondholder composition, Chari, Dovis, and Kehoe (2020), Mallucci (2022), Perez (2018), and Bolivar (2023) examine models in which the government and domestic investors do not agree on the optimal domestic demand for government bonds, so there is scope for financial repression. These papers analyze various government policies aimed at reducing the foreign debt position or increasing the share of domestic investors' savings in government bonds. They find that domestic demand for government bonds is lower compared to the centralized equilibrium, where the government directly chooses the level of foreign and domestic debt, as domestic investors fail to internalize how their demand for government bonds affects the risk of default. We propose the currency composition of debt as an alternative tool for the government to influence the domestic demand for bonds and show that when financial repression is not available, the government is willing to use it.

A key strand of the literature on optimal taxation is based on Barro (1979), which shows that governments use debt to smooth out tax distortions. Lucas and Stokey (1983b) find that under complete markets, taxes and debt follow the stochastic properties of government spending. Aiyagari et al. (2002) extend this to incomplete markets, showing that borrowing constraints introduce a near-random walk in taxes and debt. Pouzo and Presno (2022) incorporate default risk in a closed economy, leading to endogenous credit limits that restrict the government's ability to smooth shocks through debt. Our model builds on this by introducing debt currency choice and bondholder composition in an open economy with sovereign default and distortionary taxation.

Outline The remainder of the paper is organized as follows. Section 2 presents data on the composition of sovereign debt holders and their currencies for a set of emerging economies. Section 3 describes the model. Section 6 analyzes the implementation of financial repression. Section 7 presents an empirical validation of the mechanism of the model. Section 8 concludes.

2 Empirical Motivation

In this section, we document two patterns on the currency and bondholder composition of sovereign debt that motivate our analysis: (i) emerging economies issue mostly in LC, and the average share of LC debt has increased in recent decades; (ii) there is a preference among domestic investors for holding debt denominated in LC.

First, we observe that emerging economies are predominantly based on local currency. On average, 74% of their debt is denominated in their own currency. In addition, in the last decades, the average share of LC debt has seen a noticeable increase by 8 percentage points. Second, there is a distinct pattern in the currency composition of the debt held by different types of investors. Specifically, domestic investors hold, on average, close to 90% of their debt in LC.

2.1 Description of Data

Let us begin by describing the data. For debt, we use the Arslanalp and Tsuda (2014) database updated in April 2024. Our sample consists of 17 countries out of 22 in the database. We exclude Bulgaria, Chile, Colombia, Malaysia, and Mexico due to missing information on the total debt of domestic investors. Thus, we have an unbalanced panel of quarterly data for 17 emerging economies from 2004$Q1$

to 2023Q4. The countries in our sample are Argentina, Brazil, China, Egypt, Hungary, India, Indonesia, Peru, the Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine and Uruguay. This dataset contains information on central government debt securities, differentiated by type of investor and currency denomination. In particular, we can identify the amount of debt denominated in local currency (LC) and foreign currency (FC) held by domestic and foreign investors. We used the IMF IFS dataset on inflation rates to compute variables in real terms.

2.2 Currency Composition of Sovereign Debt

Table 2 presents the empirical regularities of sovereign debt by currency for 17 emerging economies. During the 20 years studied, the average level of total public debt is 49 percent of GDP. The second and third columns detail each country's share of debt denominated in local currency. On average, 74 percent of the debt issued by these countries is in local currency, indicating that emerging economies predominantly issue debt in their own currency (see Figure C.1 for the dynamics of debt in LC).

However, there is considerable heterogeneity among these economies. Although some countries exhibit a high share of LC debt, such as India, China, and Thailand, where nearly 100 percent of their sovereign debt is denominated in local currency, others fall below the average. Argentina, Ukraine, and Uruguay have local currency shares ranging from 40 to 50 percent, reflecting the lowest values within the sample.

The share of debt in local currency increased by an average of 8 percentage points across the sample. However, two countries did not see any change in their local currency debt shares for different reasons. In India, all central government debt securities have been fully denominated in local currency throughout the period. Meanwhile, in Argentina, despite considerable fluctuations over the past 20 years, the share of local currency debt has recently returned to its 2004 level.

Six countries experienced increases in their local currency debt shares above the average, with two showing significant growth. Peru and Uruguay saw the largest increases, with jumps of 55 and 72 percentage points, respectively. In contrast, five countries, including Egypt and Turkey, experienced moderate to significant declines in their local currency debt shares, with decreases of 27 and 24 percentage points, respectively. This heterogeneity in trends highlights the diverse paths these economies have followed in terms of their sovereign debt composition over the last two decades.

We highlight that the share of LC-denominated debt appears relatively low when focusing only on foreign debt. However, as shown in the second column of Table 2, once the domestic debt is included, the share of the LC debt is consistently high in all countries. In most of the countries in the sample, the LC debt accounts for more than 50% of the total debt.

2.3 Bondholder Composition of Sovereign Debt

When analyzing the debt currency composition by investor, we observe that domestic investors' holdings are mostly in local currency. On average, 89 percent of domestic debt is denominated in local currency, as shown in column 4 of Table 2. This suggests that domestic investors prefer debt denominated in their own currency.

Table 2: Empirical Regularities of Sovereign Debt by Currency

Country	Total Debt	Share of Debt in LC		Share of Domestic Debt in LC	
	Average (% of GDP)	Average (% of T. Debt)	Δ 2023 - 04 (% of T. Debt)	Average (% of Dom. Debt)	Δ 2023 - 04 (% of Dom. Debt)
Argentina	65	40	0	66	-2
Brazil	72	93	24	98	2
China	45	99	1	100	0
Egypt	80	84	-27	91	-24
Hungary	73	71	-5	95	-5
India	75	100	0	100	0
Indonesia	32	78	-19	96	-6
Peru	27	52	47	83	55
Philippines	52	74	13	86	7
Poland	51	77	2	97	-1
Romania	31	56	-18	78	-9
Russia	15	69	58	82	38
South Africa	45	91	3	100	0
Thailand	32	99	7	100	0
Turkey	37	70	-24	82	-14
Ukraine	46	49	14	90	-9
Uruguay	56	47	52	73	72
Mean	49	74	8	89	6
Median	46	74	2	91	0
Std. Dev.	19	19	25	10	25

Notes: Average total debt is calculated using total central government debt securities from 2004 till 2023. The share of debt in local currency refers to the percentage of total debt securities issued in local currency. The share of domestic debt in local currency refers to the share of debt securities issued in local currency held by domestic investors. The differences are calculated taking the share for 2023q4 and 2004q1, or the first and last observation available for each country.

In ten of the seventeen countries studied, over 90 percent of domestic debt is denominated in LC. Argentina has the lowest share, at 66 percent, while in four countries, China, India, South Africa, and Thailand, all central government debt securities held by domestic investors are fully denominated in local currency. This pattern in currency composition highlights the strong link between local currency and domestic debt. However, there is no evidence of market segmentation. As shown in Table 2, in most economies, both domestic and foreign investors hold positive amounts of bonds in LC and FC.

The dynamics of local currency debt also reveal different trajectories. On average, domestic holdings of LC debt have increased by six percentage points (see Figure C.2 for the full dynamics of domestic debt in LC). However, four countries do not present any change as their domestic debt is fully denominated in

LC. Countries such as Peru, Russia, and Uruguay have experienced significant increases in their shares of local currency debt, with 55, 38, and 72 percentage points increasing, respectively. In contrast, Egypt and Turkey have seen significant decreases in their shares, with declines of 24 and 14 percentage points, respectively. Despite these declines, most countries still maintain a relatively high share of domestic debt in local currency, with only a few showing significant downward trends.

Appendix C provides information on foreign debt. As documented by Ottonello and Perez (2019) and others, when we focus on foreign debt, we see that most of it is denominated in FC, a phenomenon known as 'original sin.' However, focusing only on foreign debt gives an incomplete picture, as governments make extensive use of LC bonds. On average, only 26 percent of total debt in these economies is issued in foreign currency. However, foreign investors hold 54 percent of their sovereign bond portfolios in FC, a share that has declined by 13 percentage points over the past two decades (see Table C.1).

Take Away. Emerging economies issue debt predominantly in local currency, and domestic investors strongly prefer to hold debt in local currency. Next, we will build a model that can account for these patterns.

3 Model

We consider a small open economy that lasts for two periods, indexed by $t \in \{0, 1\}$. The key departure from standard frameworks is the presence of two types of investors and two types of bonds. We assume that the government lacks commitment and can default on its bonds. We also assume that the law of one price applies and that the government cannot selectively default on foreign investors.

3.1 Exogenous Processes

Uncertainty is modeled by an exogenous utility cost (ϕ) that the government would face if it defaults and the exogenous exchange rate (e_1).

Assumption 1. *The stochastic structure satisfies the following conditions:*

(i) $\phi \in \mathbb{V} \equiv (\tilde{\phi}, \underline{\phi})$

(ii) ϕ *is drawn from a distribution independent of liabilities p.d.f* $f(\phi)$

(iii) the inverse of the nominal exchange rate has mean one; this is $\mathbb{E}\left[\frac{1}{e_1}\right] = 1$

(iv) $cov(e_1^{-1}, \phi) = 0$

We assume that the covariance of the nominal exchange rate and the cost of default is zero. We do so to abstract from the insurance motive that the government could have to issue LC and highlight the role of the currency composition as a determinant of the composition of bondholders and how it changes the incentives of the government to issue LC debt. We summarize the exogenous state in the second period as $s = \{e_1, \phi\}$

3.2 Domestic Investors

The representative domestic investor has an endowment of time that can be allocated to leisure or labor, denoted by n, which is used to produce goods. The investor's preferences follow the Greenwood, Hercowitz, and Huffman (1988) (GHH) framework, and the utility function is time-separable[2]. Utility over consumption $\{c_t\}$ and labor $\{n_t\}$ is represented by:

$$U = u(c_0 - v(n_0)) + \beta \mathbb{E}[u(c_1(s) - v(n_1(s))) - d(s)\phi] \tag{1}$$

where $d \in \{0, 1\}$ takes the value of 1 if the government defaults or is zero otherwise. The utility function u satisfies the following conditions: (i) $u : \mathbb{R}^2_+ \to \mathbb{R}$; and (ii) u is twice differentiable, with $u' > 0$ and $u'' < 0$. Let $\beta \in (0, 1)$ denote the discount factor.

Domestic investors are borrowing-constrained. The production function in this economy is linear, that is, $y = n$. The government finances its fiscal budget by imposing a distortionary tax on labor income.

Let $\{b_D, b_D^\star\}$ be the holdings of the domestic investor in bonds denominated in LC and FC, respectively. Also, let $\{q, q^\star\}$ be the prices of the bonds denominated in LC and FC. Then the budget constraint, expressed in FC, in the first period is:

$$\begin{gathered} c_0 + qb_D + q^\star b_D^\star = (1 - \tau_0)n_0 \\ b_D, b_D^\star \geq 0 \end{gathered} \tag{2}$$

In period one, in each state s, the budget constraint of domestic investors becomes:

$$c_1(s) = (1 - \tau_1(s))n_1(s) + (1 - d(s))\left(\frac{b_D}{e_1} + b_D^\star\right) \quad \forall s \tag{3}$$

The consumption good, c, is tradable, so the law of one price applies, $p = ep^\star$. We assume that the international price is constant and normalized to one. Thus, the domestic price level is directly related to the exchange rate, such that $p = e$. The key difference between LC bonds and FC bonds is that the real value, in period one, of LC currency bonds is linked to the nominal prices. Consequently, it depends on the realization of the inverse of the nominal exchange rate $\left(e_1^{-1}\right)$.

2. The GHH specification eliminates income effects on labor supply.

The problem of domestic investors is choosing the sequences of consumption labor and debt $\{c_0, n_0, \{c_1(s), n_1(s)\}_s, b_D, b_D^\star\}$ to maximize (1) subject to (2) and (3). The FOC of this problem are:

$$-v'(n_t) = (1-\tau_t) \qquad \forall t = 0,1 \tag{4}$$

where equation (4) is the usual optimality condition that equates the marginal cost of labor to the marginal productivity of the domestic investor after taxes. The FOC yield the following:

$$q\, u'(c_0 - v(n_0)) = \beta \mathbb{E}\left[(1-d(s))\frac{u'\Big(c_1(s) - v(n_1(s))\Big)}{e_1}\right] + \eta_1 \tag{5}$$

$$q^\star u'(c_0 - v(n_0)) = \beta \mathbb{E}\left[(1-d(s))u'\Big(c_1(s) - v(n_1(s))\Big)\right] + \eta_2 \tag{6}$$

Those are the Euler equations for the domestic investor for the bonds in LC and FC. Because domestic investors are borrowing constrained, η_1 and η_2 stand for the multipliers in these constraints. We also define the stochastic discount factor (SDF) of the domestic investors as follows:

$$\Lambda \equiv \frac{\beta u'\Big(c_1(s) - v(n_1(s))\Big)}{u'(c_0 - v(n_0))} \tag{7}$$

Finally, we define $B_D, B_D^\star$ as the aggregate positions of domestic investors in the bond market.

3.3 Foreign Investors

There is a continuum of risk-neutral foreign lenders that have access to a risk-free asset with a gross interest rate R^{-1}. Let W be the initial wealth of foreign investors. The problem of foreign investors can be written as follows:

$$\pi = \max_{\{b_F, b_F^\star\}} \; \mathbb{E}\left[(1-d(s))\left(\frac{b_F}{e_1} + b_F^\star\right)\right] + (W - qb_F - q^\star b_F^\star)\frac{1}{R} \tag{8}$$

Because the maximization problem of foreign investors is linear, we can derive the usual demand functions for government bonds.

$$b_F^\star = \begin{cases} 0 & \text{if } q^\star > \mathbb{E}\left[(1-d(s))\beta\right], \\ [0, W] & \text{if } q^\star = \mathbb{E}\left[(1-d(s))\beta\right], \\ W & \text{otherwise.} \end{cases} \tag{9}$$

$$b_F = \begin{cases} 0 & \text{if } q > \mathbb{E}\left[(1-d(s))\beta\frac{1}{e_1}\right], \\ [0, W] & \text{if } q = \left[(1-d(s))\beta\frac{1}{e_1}\right], \\ W & \text{otherwise.} \end{cases} \tag{10}$$

We also assume that they collectively have enough resources to buy any arbitrary number of bonds, so $W \to \infty$. Finally, we define B_F and $B_F^\star$ as the position of foreign debt in the economy. These piecewise-linear demands say that foreign investors are willing to buy any amount of debt if the price compensates them for the risk of default. In a model where there are only foreign investors as Eaton and Gersovitz (1981) these conditions imply that the price of FC should be equal to the probability of default. Instead, in this environment, the price could be higher in equilibrium if the domestic demand for bonds is equal to the supply[3].

3.4 Government

The government starts with an initial debt held by foreign investors, denoted as $\bar{B_0}$, and borrows B_1 and $B_1^\star$, denominated LC and FC, respectively. In addition, it collects income taxes. In period one, the budget constraint of the government expressed in FC is given by:

$$\bar{B_0} = \tau_0 n_0 + qB_1 + q^\star B_1^\star \tag{11}$$

In period two, in every state s, the fiscal budget expressed in FC is given by:

$$(1-d(s))\left(\frac{B_1}{e_1} + B_1^\star\right) = \tau_1(s)n_1(s) \tag{12}$$

3.5 Equilibrium

We are ready to define the equilibrium in the economy.

Definition 1. (Competitive Equilibrium) Given an initial debt $\bar{B_0}$ and government policies $\{B_1, B_1^\star, \tau_1, \{\tau_2(s), d(s)\}_s\}$; an equilibrium consists of a sequence of prices $\{q, q^\star\}$, and allocations $\{c_0, n_0, \{c_1(s), n_1(s)\}_s, b_D, b_D^\star\}$ such that:

i Given prices and government policies, $\{c_0, n_0, \{c_1(s), n_1(s)\}_s, b_D, b_D^\star\}$ maximizes (1) subject to (2) and (3);

ii Given government policies, foreign demand for government bonds satisfy (9) and (10);

iii Given prices and domestic investors' allocations, $\{B_1, B_1^\star, \tau_0, \{\tau_1(s), d(s)\}_s\}$, is consistent with government budget constraints (11) and (12);

3. Domestic investors are risk-averse while foreign lenders are risk neutral.

iv Markets clear: $B_D = B_1 - B_F$, $B_D^\star = B_1^\star - B_F^\star$, $B_D = b_D$ and $B_D^\star = b_D^\star$.

Equilibrium in the Labor Market. Before analyzing the optimal policy, we derive some equilibrium conditions in this economy. First, to characterize the equilibrium in the labor market, we combine the budget constraint of the government and the FOC of the labor supply to derive:

$$-v'(n_0) = \left[1 - \frac{\bar{B}_0 - qB_1 - q^\star B_1^\star}{n_0}\right] \tag{13}$$

$$-v'(n_1(s)) = \left[1 - \frac{(1-d(s))\left(\frac{B_1}{e_1} + B_1^\star\right)}{n_1(s)}\right] \tag{14}$$

These equilibrium conditions show that the government uses debt, $B_1^\star$ and B_1, to smooth the tax distortions in the labor market[4]. On the one hand, higher debt reduces the distortion in the labor market in the first period, but on the other hand, it increases the distortion in the labor market in the second period in states of repayment. Moreover, if the government issues debt denominated in LC, the size of the distortion in the labor market in the second period depends on the realization of the inverse of the nominal exchange rate.

Resource Constraints. Finally, we can use the budget constraint of the domestic investor and the government to derive the resource constraints in the economy.

$$c_0 + \bar{B}_0 = n_0 + q(B_1 - B_D) + q^\star(B_1^\star - B_D^\star) \tag{15}$$

$$c_1 = n_1(s) - (1-d(s))\left(\frac{B_1 - B_D}{e_1} + B_1^\star - B_D^\star\right) \tag{16}$$

Note that the resource constraints depend on the foreign debt position in LC and FC, as well as on production, which is affected by the size of the government debt through distortionary taxes.

4 Optimal Policy

In this section, we characterize the optimal policy of the government. The preferences of the government are given by:

$$U = u\big(c_0 - v(n_0)\big) + \beta\mathbb{E}\Big[u\big(c_1(s) - v(n_1(s))\big) - d(s)\phi\Big] \tag{17}$$

4. See Pouzo and Presno (2022), for a recent analysis of tax smoothing in models of sovereign default.

We assume that the government is as impatient as domestic investors, discounting utility by β. We solve the government's problem using backward induction: first deriving the optimal policy in period one, and then using that result to determine the optimal policy in period zero.

4.1 Default Function

We define the aggregate state on the bond market as $\boldsymbol{B} \equiv (B_1^{\star}, B_1, B_D^{*}, B_D)$. The relevant state of the economy in period one is the state in the bond market ($\boldsymbol{B}$), and the exogenous state (s). The government solves the following.

$$V_1(\boldsymbol{B}, s) = \max_{d \in \{0,1\}} (1-d)V^R(\boldsymbol{B}, e_1) + dV^D(\phi) \tag{18}$$

Here V^R and V^D are the values of repayment and default, respectively. The value of repayments is given by the following:

$$V^R(\boldsymbol{B}, e_1) = u\left(n - \frac{B_1 - B_D}{e_1} - B_1^{\star} + B_D^{\star} - v\,(n)\right) \tag{19}$$
$$\textit{subject to}$$
$$-\,v'(n) = \left[1 - \frac{\frac{B_1}{e_1} + B_1^{\star}}{n}\right]$$

The constraint stands for the government's implementability constraint in the labor market. The value of repayment depends on the real value of the government debt, which in turn depends on the foreign position of the economy and the realization of the nominal exchange rate, as both affect the resource constraint. The value of default is:

$$V^D(\phi) = u\,(n - v\,(n)) - \phi \tag{20}$$
$$\textit{subject to}$$
$$-\,v'(n) = 1$$

The government's only choice in the second period is whether to default or to repay. We characterize the default decision of the government by defining the following threshold.

$$V^D\left(\hat{\phi}(\boldsymbol{B}, e_1)\right) = V^R(\boldsymbol{B}, e_1) \tag{21}$$

This threshold represents the utility surplus derived from the higher consumption under default. Using this threshold, the default function of the government is

$$D(\boldsymbol{B}, e_1) = \begin{cases} 1 & \text{if} \quad \hat{\phi}(\boldsymbol{B}, e_1) > \phi \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

The government defaults in the second period if the default cost is less than the surplus from higher consumption. Using the default function of the government, the probability of default conditional on the realization of the nominal exchange rate can be written as

$$F\left(\hat{\phi}(\boldsymbol{B}, e_1)\right) = \int_{\underline{\phi}}^{\hat{\phi}(\boldsymbol{B}, e_1)} f(\phi) d\phi \tag{23}$$

Price Functions. The conditional probabilities of default and the demand for government bonds by foreign and domestic investors determine the price functions. The price schedules of government bonds are as follows:

$$Q^{\star}(\boldsymbol{B}) = \begin{cases} \mathbb{E}\left[(1 - F_{\phi}(\hat{\phi}(\boldsymbol{B}, e_1))\beta\right] & \text{if } B_F^{\star} > 0 \\ \mathbb{E}\left[(1 - F_{\phi}(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\right] & \text{if } B_F^{\star} = 0 \end{cases} \tag{24}$$

$$Q(\boldsymbol{B}) = \begin{cases} \mathbb{E}\left[(1 - F_{\phi}(\hat{\phi}(\boldsymbol{B}, e_1))\beta\frac{1}{e_1}\right] & \text{if } B_F > 0, \\ \mathbb{E}\left[(1 - F_{\phi}(\hat{\phi}(\boldsymbol{B}e_1))\Lambda(\boldsymbol{B}, e_1)\frac{1}{e_1}\right] & \text{if } B_F = 0 \end{cases} \tag{25}$$

Where $\Lambda(\boldsymbol{B}, e_1)$ is the SDF of the domestic investors in each state as defined in (7). There are two possible equilibrium conditions for the price of FC (or LC) bonds. The first is when foreign investors participate in the market, so $B_F^{\star} > 0$ (or $B_F > 0$). In this case, they are the marginal investors, and the price of the bond equals the probability of default discounted by β, as in the standard model (the price of the bond denominated in LC is also discounted by the inverse of the nominal exchange rate, e_1^{-1}). The second case is where $B_F^{\star} = 0$ (or $B_F = 0$). In this case, foreign investors do not participate in the market and the marginal investors are the domestic investors, so the SDF of domestic investors is used to price the bond.

4.2 Domestic Demand

We can now study the equilibrium in period zero. We begin by analyzing the problem of domestic investors. Using the price function and the optimality conditions of the domestic investors (6) and (5), we characterize their problem as follows:

Lemma 1. *Given debt policy $B_1^{\star}, B_1$, the portfolio $B_D^{\star}, B_D$ solves the problem of domestic investors if and only if:*

$$0 = \eta_1\Big(Q(\boldsymbol{B}) - \mathbb{E}\Big[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\frac{\Lambda(\boldsymbol{B}, e_1)}{e_1}\Big]\Big) \tag{26}$$

$$0 = \eta_2\Big(Q^\star(\boldsymbol{B}) - \mathbb{E}\big[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\big]\Big) \tag{27}$$

$$B_D = 0 \quad if \quad \eta_1 > 0. \tag{28}$$

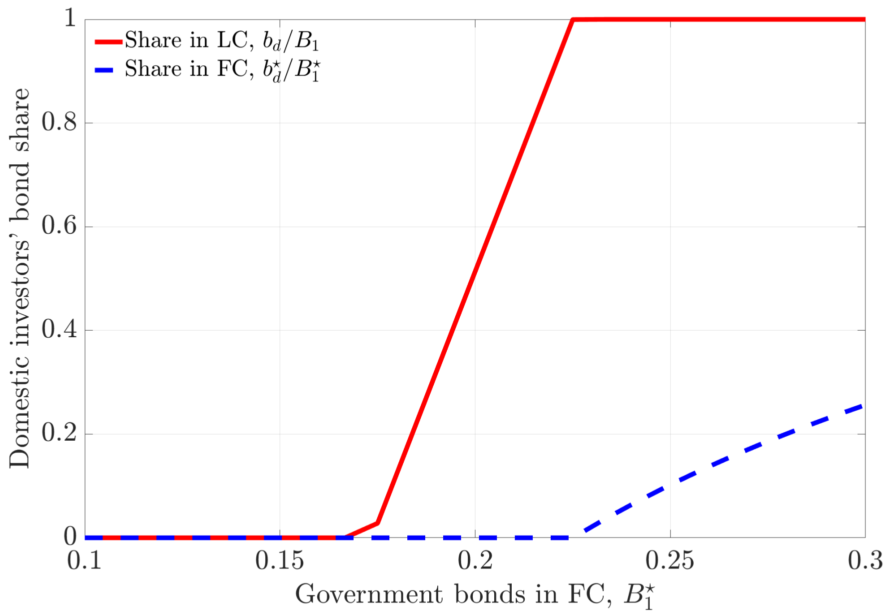
$$B_D^\star = 0 \quad if \quad \eta_2 > 0. \tag{29}$$

Proof. The proof is given in Appendix A.1 □

The lemma 1 establishes that the Euler equation of domestic investors is a necessary and sufficient condition to solve the problem of domestic investors for a given government debt policy. Given prices and the probability of default, the problem of the government and the problem of domestic investors is the maximization of a concave utility function subject to a convex set. As a result, the solution to the problem is unique.

Figure 1 shows domestic demand for government bonds at a fixed level of LC debt and varying levels of FC issuance. When total government debt is low, domestic investors do not hold any bonds. As the government issues FC bonds, domestic investors gradually increase their holdings of LC bonds until they absorb the entire stock of LC debt. With further increases in FC debt, they begin to demand FC bonds as well. In sum, as total debt rises, domestic investors first absorb the full amount of LC debt and then start holding FC bonds.

Figure 1: Domestic Demand for Government Bonds



Notes: This figure displays the domestic demand for government bonds in LC and FC. The solid red line corresponds to the ratio of demand for LC bonds-to-LC bonds issuance, and the dashed blue line corresponds to the ratio of demand for FC bonds-to-FC bonds issuance. We use the following functional forms: $u(x) = \frac{x^{1-1/\sigma}}{1-1/\sigma}$ where $x = c - \psi \frac{n^{1+1/\eta}}{1+1/\eta}$. The parameters are: $\beta = 0.95, \sigma = 0.5, \eta = 1, \psi = 0.1$ and $B_0 = 0.4$.

The intuition of the result can be understood as follows. The only source of uncertainty for domestic investors in the model is the level of taxes in the second period. When the government chooses to issue debt, domestic investors increase savings, increasing resources in states where the government will choose to repay, and consequently, taxes are high. Because government bonds are paid in states where taxes are high, they provide good insurance for domestic investors.

In addition, when the government issues debt denominated in LC, taxes become more volatile because the value of the government debt changes with the realization of the nominal exchange rate. To get insurance against this risk, domestic investors buy government bonds denominated in LC that are highly valued in states where taxes are particularly high. As a result, domestic investors have an additional incentive to buy LC bonds to isolate consumption from shocks to the nominal exchange rate. As a result, when the government increases the supply of bonds denominated in FC, domestic investors first increase their demand for LC bonds and they buy FC bonds only when the hold 100% of LC debt.

In summary, the optimal policy for domestic investors is to buy bonds denominated in LC until the government's total debt is high enough that they demand a hundred percent of government bonds denominated in LC. We formalize this result in the following proposition.

Proposition 1. *Assume $Cov(e_1^{-1}, \phi) = 0$. Let $B_1^\star > 0$, $B_1 \geq 0$. Then $B_D^\star > 0$ if and only if $B_D = B_1$.*

Proof. The proof is given in Appendix A.2 □

Proposition 1 formalizes the main results of this section. Domestic investors tend to buy more bonds denominated in LC. The LC bond provides a better insurance for the domestic investors compared to the FC bond because the real value of LC bonds has a stronger correlation with taxes. As a result, domestic investors prefer to buy LC bonds and only demand FC bonds when they buy the full stock of LC debt.

In order to abstract for any insurance motive that the household or the government might have to use debt denominated in LC we assume $\text{Cov}(e_1^{-1}, \phi) = 0$. This allows us to highlight the new mechanism in our model, which is that LC debt has a higher correlation with taxes which, in turn, generates a bias on domestic investors towards those bonds. Instead, when there is a covariance of the nominal exchange rate with the fundamentals, there is a second effect which is that default could be positively correlated with the nominal exchange rate so the total correlation of the nominal exchange rate and taxes depends on the parameters. To be clear, under the assumption that $\text{Cov}(e_1^{-1}, \phi) = 0$ households first buy the full stock of LC bonds before increasing their demand for FC bonds. This is a consequence that under this assumption $B_1 \geq 0$ increase risk. On the other hand, when $\text{Cov}(e_1^{-1}, \phi) \leq 0$ there could be positive domestic demand for FC while $B_D < B_1$ if B_1 is low enough.

We close this section by defining $\mathcal{B}_D^\star(B_1, B_1^\star)$ and $\mathcal{B}_D(B_1, B_1^\star)$, which are the functions that determine the domestic debt of the economy consistent with the optimality conditions of households summarized in Lemma 1 for each government debt policy.

4.3 Optimal Government Debt

The government's problem in period zero is to solve:

$$
\begin{aligned}
V_0 = & \max_{B_1, B_1^\star} u\big(c_0 - v(n)\big) + \beta \mathbb{E}\big[V_1(\boldsymbol{B}, s)\big] \\
& \textit{subject to} \\
& c_0 + \bar{B}_0 = n + Q(\boldsymbol{B})(B_1 - B_D) + Q^\star(\boldsymbol{B})(B_1^\star - B_D^\star) \\
& - v'(n) = \left[1 - \frac{\bar{B}_0 - Q(\boldsymbol{B})B_1 - Q^\star(\boldsymbol{B})B_1^\star}{n}\right] \\
& Q^\star(\boldsymbol{B}), \;\; Q(\boldsymbol{B}) \\
& B_D = \mathcal{B}_D^\star(B_1, B_1^\star), \;\; B_D^\star = \mathcal{B}_D(B_1, B_1^\star)
\end{aligned}
\tag{30}
$$

The government's problem in the first period is choosing a debt policy subject to implementability constraints in the labor and bond markets. In the bond market, the government must satisfy the optimal conditions of the domestic and foreign investors represented in the price functions and in the domestic debt position function derived from the problem of the domestic investors. The FOC of the government yields the Generalized Euler Equation (GEE). For expositional convenience, we focus on the GEE for the LC-denominated. Which we characterize in the following Proposition:

Proposition 2. *Let N_0 and N_1 be the equilibrium labor supply in period zero and period one under repayment. Then the GGE of the government in LC is:*

$$\underbrace{\left(\frac{\partial N_0}{\partial B_1}\right)\left(1-\frac{\partial v_0}{\partial B_1}\right)=\mathbb{E}\left[\left(1-F_\phi(\hat{\phi}(\boldsymbol{B},e_1))\right)\Lambda(\boldsymbol{B},e_1)\frac{\partial N_1^R}{\partial B_1}\left(\frac{\partial v_1}{\partial B_1}-1\right)\right]}_{\textit{Tax Smooth}}$$

$$-\underbrace{\left(\frac{\partial Q}{\partial B_1}\right)(B_1-B_D)-\left(\frac{\partial Q^\star}{\partial B_1}\right)(B_1^\star-B_D^\star)}_{\textit{Default Risk}}-\underbrace{\omega(B,B^\star)\frac{\partial B_D}{\partial B_1}-\omega^\star(B,B^\star)\frac{\partial B_D^\star}{\partial B_1}}_{\textit{Financial Distortion}}$$

Where $\frac{\partial \mathcal{B}_D^\star}{\partial B_1}$ and $\frac{\partial \mathcal{B}_D}{\partial B_1}$ stand for the change in the domestic debt position, in FC and LC. In addition, $\omega(B_1,B_1^\star)$ and $\omega^\star(B_1,B_1^\star)$ are defined as:

$$\omega(B,B^\star)=\left(\frac{\partial Q}{\partial B_D}\right)(B_1-B_D)+\left(\frac{\partial Q^\star}{\partial B_D}\right)(B_1^\star-B_D^\star)+\left(\frac{\partial N_0}{\partial B_D}\right)\left(1-\frac{\partial v_0}{\partial B_1}\right)$$

$$\omega^\star(B,B^\star)=\left(\frac{\partial Q}{\partial B_D^\star}\right)(B_1-B_D)+\left(\frac{\partial Q^\star}{\partial B_D^\star}\right)(B_1^\star-B_D^\star)+\left(\frac{\partial N_0}{\partial B_D^\star}\right)\left(1-\frac{\partial v_0}{\partial B_1}\right)$$

Proof. The proof is in Appendix B □

Proposition 2 characterizes the main trade-off of the government when choosing debt. In the absence of financial frictions, the government's optimality condition is to equalize the marginal cost of distortions in the labor market between periods zero and one. In contrast, in the GEE the government deviates from tax smoothing for two reasons. First, due to default risk, the GEE accounts for the fact that the labor market is distorted in period one only in states of repayment, and it also considers that increasing debt affects bond prices[5].

Second, a wedge exists between the government and domestic investors regarding the optimal level of foreign debt. This wedge accounts for deviations in domestic demand for government bonds from the optimal level from the perspective of the government. Domestic investors do not internalize that increasing their demand for government bonds reduces foreign debt, decreases the probability of default in period one, and increases bond prices. If the choice of domestic investors was to coincide with the optimal demand from the government's perspective, we could use the envelope theorem to cancel the effect of higher government debt on the foreign debt position.

On the other hand, the government acts as a monopolist and internalizes the effect of their policies on bond prices. In particular it internalizes that different debt policies lead to different levels of domestic demand, which in turn affect bond prices. As a result, due to this wedge over optimal demand, the

5. See Pouzo and Presno (2022) for a related result.

government's GEE reflects the cost for the government of changes in the domestic demand for bonds.

To further characterize how the wedge between government and domestic investor affects optimal policy, we determine the sign of the wedge. In particular, we establish:

Lemma 2. *Assume* $B_1 > 0$ *or* $B_1^\star > 0$*, then* $\omega(B_1, B_1^\star) > 0$ *and* $\omega^\star(B_1, B_1^\star) > 0$*.*

Proof. The proof is given in Appendix A.3 □

This result effectively extends the result in Bolivar (2023), among others, who established a similar result in an environment without tax distortions. This friction plays an important to understand the role of the currency composition of the sovereign debt.

4.4 Equilibrium.

We are ready to define the Markov equilibrium in this environment.

Definition 2. (Markov Equilibrium) Given initial debt $\bar{B}_0$, a Markov equilibrium is a set of value functions $V_0, V_1(\boldsymbol{B}, s)$, price functions $Q(\boldsymbol{B}), Q^\star(\boldsymbol{B})$, and policy functions $\mathcal{B}_D^\star(B_1, B_1^\star), \mathcal{B}_D(B_1, B_1^\star), \mathcal{B}, \mathcal{B}^\star$ such that:

(i) given the price, $\{\mathcal{B}_D^\star(B_1, B_1^\star), \mathcal{B}_D(B_1, B_1^\star)\}$ solves the domestic investor's problem at every state;

(ii) $Q(\boldsymbol{B}), Q^\star(\boldsymbol{B})$ satisfy (25) and (24);

(iii) $\mathcal{D}, \mathcal{B}, \mathcal{B}^\star$ solve the government problem at every state, and V_0, V_1 attain the maximum

4.5 Optimal Currency Composition

Given the equilibrium conditions, it is possible to establish.

Proposition 3. *Assume* $Cov(e_1^{-1}, \phi) = 0$*. Let* $B_1^\star > 0$*. Then* $B_1 = 0$ *cannot be part of the Markov equilibrium.*

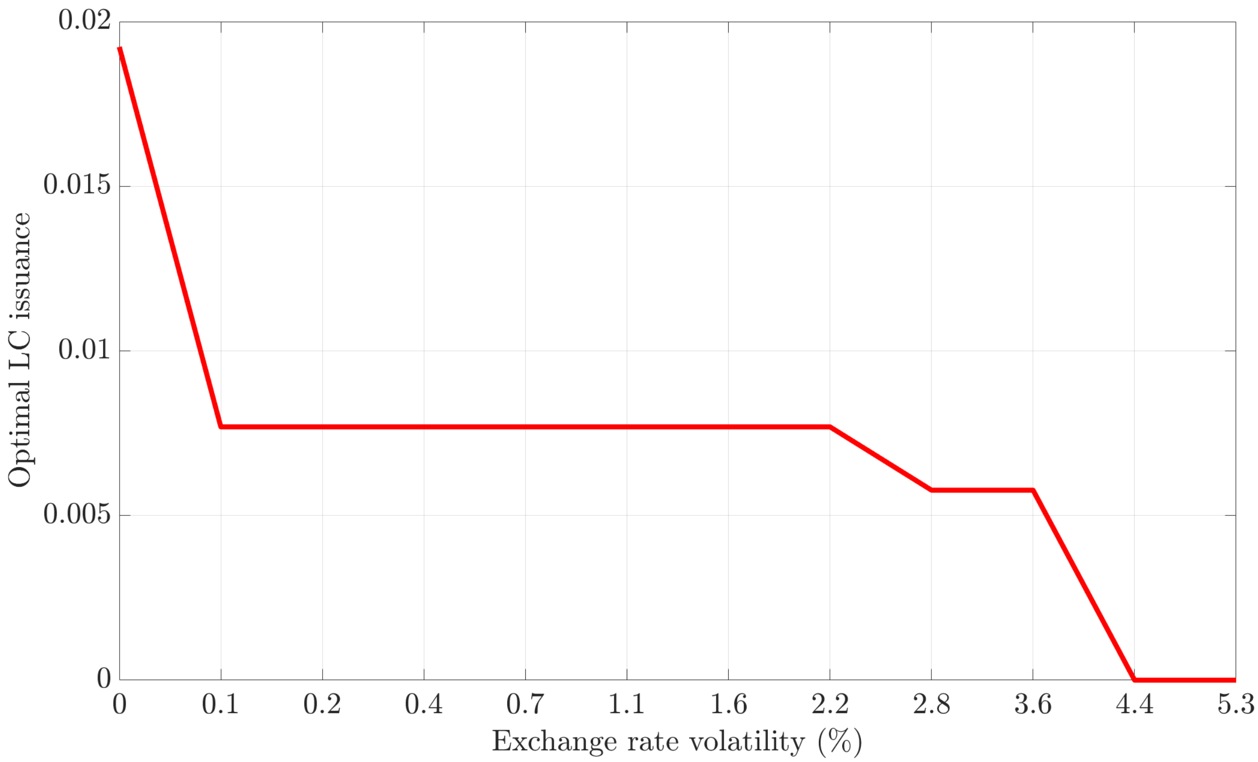
Proof. The proof is given in Appendix A.4 □

Proposition 3 establishes a key result of the model. Even if the stochastic structure is such that the government cannot get any insurance for issuing debt denominated in LC, it still has incentives to use LC debt as an active tool. There are two key conditions for this result. First, domestic investors have stronger incentives to buy LC-denominated debt than FC-denominated debt as establish in Proposition 1. Second, there is a wedge in the domestic demand for government bonds between the optimal choice of the domestic investors and the government as establish in Proposition 2. Under these conditions, the government uses the LC as a tool to reduce the position of the economy's foreign debt.

To understand the main trade-off of the government regarding FC bond in Figure 2 we plot the optimal supply of LC bonds as a function of the volatility of the nominal exchange rate.

In the model, there is no role for insurance of LC debt. As the government increases the supply for LC bond it increases volatility of default and tax policies. From the perspective of the government it has a cost in terms of utility as the government inherits the risk aversion of the domestic investors. However it also generates incentives to domestic investors to increase their demand for government bonds which closes the wedge in the domestic debt. For low and intermediate levels of the volatility of the nominal exchange rate the benefits of using LC are higher and the government chooses to issue LC bonds. As the volatility of the nominal exchange rate increases the cost of using LC as a tool to reduce the wedge in the bond market becomes larger so the government chooses to use less LC bonds[6].

Figure 2: Optimal LC Issuance



Notes: To compute this figure we use the following functional forms: $u(x) = \frac{x^{1-1/\sigma}}{1-1/\sigma}$ where $x = c - \psi \frac{n^{1+1/\eta}}{1+1/\eta}$. The parameters are: $\beta = 0.95, \sigma = 0.5, \eta = 1, \psi = 0.1$ and $B_0 = 0.4$.

6. Numerically the optimal supply of LC bonds goes to zero, however from Proposition 3 we know LC will asymptotically go down as the volatility of the nominal exchange rate increases

5 Constrained Efficient Equilibrium

Consider a planner who chooses the domestic demand for government bonds in LC and FC. The foreign lenders' problem and price schedules remain unchanged. The planner's problem in $t = 1$ is the same as the government's problem in the second period. In the first period, the planner solves:

$$\begin{aligned} V_0^{CEE} = & \max_{B_1, B_1^\star, B_D, B_D^\star} u(c_0 - v(n)) + \beta \mathbb{E}[V_1(\boldsymbol{B}, s)] \\ & \textit{subject to} \\ & c_0 + \bar{B}_0 = n + Q(\boldsymbol{B})(B_1 - B_D) + Q^\star(\boldsymbol{B})(B_1^\star - B_D^\star) \\ & - v'(n) = \left[1 - \frac{\bar{B}_0 - Q(\boldsymbol{B})B_1 - Q^\star(\boldsymbol{B})B_1^\star}{n}\right] \\ & Q(\boldsymbol{B}), \quad Q^\star(\boldsymbol{B}) \end{aligned} \tag{31}$$

The planner chooses the debt policy and the domestic debt allocations subject to the resource constraint, the implementability condition in labor, and the price schedules. In the bond market, prices are pinned down by domestic and foreign lenders' optimality conditions, so the planner takes them as given.

We are ready to define the equilibrium in the economy.

Definition 3. (Recursive Constrained Efficient Equilibrium) A recursive constrained efficient equilibrium consists of a set of policy functions $\mathcal{B}_D^\star, \mathcal{B}_D, \mathcal{B}, \mathcal{B}^\star$, set of value functions $V_0^{CEE}, V_1(\boldsymbol{B}, s)$, and price functions $Q(\boldsymbol{B}), Q^\star(\boldsymbol{B})$ such that:

(i) The policy functions solve the planner's recursive problem and $V_0^{CEE}, V_1(\boldsymbol{B}, s)$ attain the maximum

(ii) Price functions are consistent with the demand function of foreign investors and the Euler equations of the domestic investors.

The FOCs with respect to B_1 and $B_1^\star$ reproduce the GEE from the decentralized problem. In a constrained efficient economy, it is possible to use the optimality conditions of B_D and $B_D^\star$ to establish:

Lemma 3. *The portfolio $\{B_D^\star, B_D\}$ solves the problem of the planner in a constrained efficient economy if and only if:*

$$0 = \eta_1^{CEE} \left[\left(\frac{\partial Q}{\partial B_D} \right) (B_1 - B_D) + \left(\frac{\partial Q^\star}{\partial B_D} \right) (B_1^\star - B_D^\star) + \left(\frac{\partial N_0}{\partial B_D} \right) \left(1 - \frac{\partial v_0}{\partial B_1} \right) \right]$$

$$0 = \eta_2^{CEE} \left[\left(\frac{\partial Q}{\partial B_D^\star} \right) (B_1 - B_D) + \left(\frac{\partial Q^\star}{\partial B_D^\star} \right) (B_1^\star - B_D^\star) + \left(\frac{\partial N_0}{\partial B_D^\star} \right) \left(1 - \frac{\partial v_0}{\partial B_1} \right) \right]$$

$$B_D = 0 \quad if \quad \eta_1^{CEE} > 0$$

$$B_D^\star = 0 \quad if \quad \eta_2^{CEE} > 0$$

Lemma 3 establishes that the planner's choice of domestic debt must satisfy the domestic investors' budget constraints. These FOCs match the wedges in the GEE of the decentralized economy. The planner internalizes how the domestic demand affects the government borrowing terms and thus chooses B_D and $B_D^\star$ to set the wedges to zero. Therefore:

Proposition 4. *The decentralized Markov equilibrium is not constrained efficient.*

Comparing the optimal domestic demand in both problems, we can see that in the decentralized economy, domestic demand is inefficiently low. The FOC of b_D when local currency debt is held by domestic investors satisfies:

$$Q(\boldsymbol{B}) = \mathbb{E} \left[\left(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1)) \right) \frac{\Lambda(\boldsymbol{B}, e_1)}{e_1} \right]$$

Whereas in the constrained efficient allocation:

$$Q(\boldsymbol{B}) = \mathbb{E} \left[\left(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1)) \right) \frac{\Lambda(\boldsymbol{B}, e_1)}{e_1} \right] + \left(\frac{\partial N_0}{\partial B_D} \right) (1 - v_0') + \left(\frac{\partial Q}{\partial B_D} \right) (B_1 - B_D) + \left(\frac{\partial Q^\star}{\partial B_D} \right) (B_1^\star - B_D^\star)$$

Domestic investors do not internalize the effect of their demand on government borrowing terms. The second term on the right-hand side captures the effect on labor, while the last term reflects the feedback on prices. Higher domestic demand lowers the probability of default and raises bond prices. The planner accounts for these effects, whereas investors in the decentralized economy do not.

This result is related to Mallucci (2022) and Bolivar (2023), who also show that domestic demand is inefficiently low. Similarly to our environment, in both cases, the inefficiency arises because investors do not internalize how their demand affects the government's borrowing terms.

Since the planner chooses both the total amount of debt and the domestic demand of government bonds, the trade-off between LC and FC debt that exists in the decentralized economy disappears. There is no longer a gain from issuing LC debt to increase domestic demand, as the planner directly selects domestic bond holdings. In the model, higher volatility of the nominal exchange rate e_1 increases the marginal cost of LC debt for three reasons. First, it amplifies labor distortions in repayment states: when e_1 is low, the real value of LC repayments B_1/e_1 is high, requiring greater labor effort and increasing the marginal disutility of taxation. Second, it raises the probability of default by increasing the fiscal cost of repayment. Third, it heightens the sensitivity of bond prices to nominal exchange rate risk. Since LC debt is state-contingent, greater volatility in e_1 raises $\mathbb{E}[1/e_1]$ via Jensen's inequality, lowering the bond price. In contrast, FC debt implies a constant real repayment and leads to smoother distortions and default incentives. As a result, the planner prefers to issue FC debt when the volatility of e_1 is high.

6 An Economy with Financial Repression

To highlight the role of inefficiency in the foreign debt position in the economy, we now consider a government that can control the domestic demand for FC-denominated government bonds. Following Chari, Dovis, and Kehoe (2020); we consider a government that is able to impose a minimum requirement on these bonds through *Financial Repression.*

6.1 A Minimum Requirement for Domestic Investors

In a regulated economy, domestic investors choose labor and consumption as in the baseline, but face a new constraint on holdings of government bonds denominated in FC. In particular, the financial constraints of domestic investors in period one become:

$$\begin{aligned} b_D^\star &\geq \Phi B_1^\star \\ b_D &\geq 0 \end{aligned} \tag{32}$$

The government incurs no cost from imposing this minimum requirement, so its budget constraint remains unchanged. The problem of foreign investors remains unchanged. With this change, a competitive equilibrium in a regulated economy is defined as follows.

Definition 4. (Regulated Competitive Equilibrium) Given an initial debt level $\bar{B}_0$ and government policies $\{B_1, B_1^\star, \tau_1, \{\tau_2(s), d(s)\}_s, \Phi\}$, an equilibrium consists of a sequence of prices $\{q, q^\star\}$, and domestic investors' allocations $\{c_0, n_0\{c_1(s), n_1(s)\}_s, b_D, b_D^\star\}$ such that

i Given prices and government policies, $\{c_0, n_0\{c_1(s), n_1(s)\}_s, b_D, b_D^\star\}$ maximizes (1) subject to (2), (3) and (32);

ii Given government policies, foreign demand for government bonds satisfy (9) and (10);

iii Given prices and domestic investors' allocations, $\{B_1, B_1^\star, \tau_1, \{\tau_2(s), d(s)\}_s\}$, is consistent with government budget constraints (11) and (12);

iv Markets clear: $B_D = B_1 - B_F$, $B_D^{\star} = B_1^{\star} - B_F^{\star}$, $B_D = b_D$ and $B_D^{\star} = b_D^{\star}$.

In a regulated economy, domestic investors have to meet one additional constraint, while the government has an additional policy instrument (Φ).

Equilibrium in the Bond Market. The government can force domestic investors to maintain a minimum requirement of government debt for any individual investor, but it cannot manipulate aggregate prices.

6.2 Optimal Policy

Next, we analyze how optimal government policy changes when it has access to *Financial Repression*. Note that the optimal policy in period two remains unchanged, so we analyze the optimal policy in period zero.

Problem of Domestic Investors. In a regulated economy, it is possible to use the optimality conditions of domestic investors to establish:

Lemma 4. *Given debt policy $\{B_1^{\star}, B_1, \Phi\}$, the portfolio $\{B_D^{\star}, B_D\}$ solves the problem of the domestic investors in a regulated economy if and only if:*

$$0 = \eta_1^{Reg}\Big(Q(\boldsymbol{B}) - \mathbb{E}\left[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\frac{\Lambda(\boldsymbol{B}, e_1)}{e_1}\right]\Big) \tag{33}$$

$$0 = \eta_2^{Reg}\Big(Q(\boldsymbol{B}) - \mathbb{E}\left[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\right]\Big) \tag{34}$$

$$B_D = 0 \quad if \quad \eta_1^{Reg} > 0. \tag{35}$$

$$B_D^{\star} = \Phi B_1^{\star} \quad if \quad \eta_2^{Reg} > 0. \tag{36}$$

$$\tag{37}$$

Proof. The proof is given in Appendix A.5 □

The key result of this analysis is that if the minimum requirement is met, the Euler equations of domestic investors should not hold for debt denominated in FC. As a result, the government can directly determine the demand for government bonds by adjusting the minimum requirement. We define $\mathcal{B}_D^{Reg}(B_1, B_1^{\star})$ as domestic debt functions in a regulated economy.

Problem of the Government. The problem of the government in a regulated economy becomes:

$$
\begin{aligned}
V_0^{Reg} = \max_{B_1, B_1^\star, B_D^\star} & \; u(c_0 - v(n)) + \beta \mathbb{E}[V_1(\boldsymbol{B}, s)] \\
& \textit{subject to} \\
& c_0 + \bar{B}_0 = n + Q(\boldsymbol{B})(B_1 - B_D) + Q^\star(\boldsymbol{B})(B_1^\star - B_D^\star) \\
& -v'(n) = \left[1 - \frac{\bar{B}_0 - Q(\boldsymbol{B})B_1 - Q^\star(\boldsymbol{B})B_1^\star}{n} \right] \\
& Q^\star(\boldsymbol{B}), \;\; Q(\boldsymbol{B}), \;\; \mathcal{B}_D^{Reg}(B_1, B_1^\star)
\end{aligned} \tag{38}
$$

Note that we dropped the implementability constraint of the domestic demand for FC-denominated bonds as a government problem. We solve for the optimal allocations by choosing the total debt in FC and LC and the economy's domestic debt position in FC. We then use the optimal $B_D^\star$ to derive the minimum requirement that implements these allocations.

The FOC of the government for foreign debt yields: $\omega^\star(B_1, B_1^\star) = 0$. That is, the government uses *Financial Repression* to close the wedge in the domestic demand for government bonds denominated in FC. This is a direct result of the envelope theorem.

Equilibrium. We define the Markov equilibrium in this environment as follows.

Definition 5. (Markov Equilibrium of a Regulated Economy) Given initial debt $\bar{B}_0$, a Markov equilibrium is a set of value functions $V_0^{Reg}, V_1(\boldsymbol{B}, s)$, price functions $Q(\boldsymbol{B}), Q^\star(\boldsymbol{B})$, and policy functions $\mathcal{B}_D^{Reg}(B_1, B_1^\star), \mathcal{B}_D^\star(B_1, B_1^\star), \mathcal{B}, \mathcal{B}^\star$ such that:

(i) given the price, $\{\mathcal{B}_D^{Reg}(B_1, B_1^\star)\}$ solves the domestic investor's problem at every state;

(ii) $Q(\boldsymbol{B}), Q^\star(\boldsymbol{B})$ satisfies (25) and (24);

(iii) $\mathcal{B}_D^\star, \mathcal{B}, \mathcal{B}^\star$ solves the government problem at every state, and V_0, V_1 attains the maximum

It is possible to establish that the government does not use LC debt in the regulated economy. Formally;

Proposition 5. *Assume $Cov(e_1^{-1}, \phi) = 0$. Let $B_1^\star > 0$. Then $B_1 = 0$ is part of the Markov equilibrium in a regulated economy.*

Proof. The proof is given in Appendix A.6 □

Proposition 5 establishes that the government does not use LC if two conditions hold. First, LC does not provide insurance. Second, there is no wedge in the domestic demand for bonds denominated in

FC.

7 Inspecting the Mechanism

In this section, we explore the empirical validation of some of the main mechanisms of the model.

7.1 Total and Domestic Debt in the Data

A key prediction of our theory is that an increase in the supply of LC bonds should be positively correlated with the domestic demand for LC bonds. In particular, the model implies a stronger positive association between the supply of LC bonds and their domestic demand compare the correlation of the supply of FC bonds and their domestic demand.

We focus on the relationship between the growth in the total stock of public debt and the corresponding changes in domestic investors' holdings of local and foreign-denominated debt. The following regressions aim to measure this relationship for both types of currency, following the approach of Broner et al. (2022b).

$$\triangle B_{it}^{D} = \gamma_1 + \gamma_2 \triangle B_{it} + \gamma_3 X_{it-1}^{D} + \gamma_4 X_{it-1}^{D} \triangle B_t + \nu_t \qquad \forall i \in LC, FC$$

where $\triangle B_{it}^{D} = B_{it}^{D} - B_{it-1}^{D}$ denotes the change in domestic debt in LC or FC, $\triangle B_{it} = B_{it} - B_{it-1}$ denotes the change in total debt in LC or FC and $X_{it-1}^{D} = B_{it-1}^{D} / B_{it-1}$ denotes the average domestic share of LC or FC.

Table 2 presents the results of the regression for the same panel of 17 EM over a 20-year period. An increase in the government's total stock of LC debt is associated with a larger increase in domestic investors' holdings of LC debt than in their holdings of FC debt when the government issues FC debt. This empirical result supports the intuition of the model.

Table 3: Domestic Demand of Total Debt by Currency

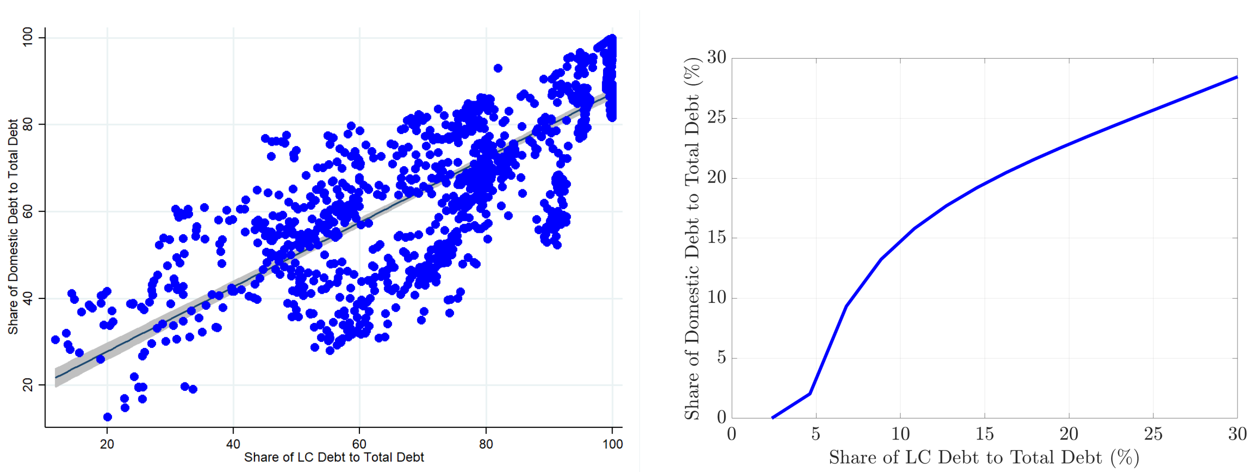
	△ Domestic Debt in LC	△ Domestic Debt in FC
△ *Total debt*$_{it}$	0.599***	0.753***
	(0.053)	(0.146)
$Share^{D}_{it-1}$	-0.007	-0.097
	(0.090)	(0.064)
△ *Total debt*$_{it}$ * $Share^{D}_{it-1}$	0.403***	-7.406***
	(0.055)	(1.415)
Time dummies	Yes	Yes
Country fixed effects	Yes	Yes
Observations	705	619

Notes: Domestic Debt in LC (FC) and Total Debt in LC (FC) are measured in real terms with a fixed exchange rate in percent of GDP. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

7.2 Currency and Bondholder Composition

Next, we explore a second key prediction of the model. In our framework, the government can use the currency composition of the debt to influence the composition of the bondholders. Figure 3 plots the relationship between the currency composition and the composition of the bondholders, in both the data and the model.

Figure 3: Currency and Bondholder Composition



(a) Data

(b) Model

Panel 3b shows that in the model, an increase in the share of LC debt leads domestic investors to increase their savings, resulting in a higher share of domestic debt. Similarly, panel 3a plots this relationship in the data and confirms a strong positive correlation between the share of LC debt and the share of domestic debt, consistent with the model.

8 Conclusion

This paper examines the relationship between the currency composition of sovereign debt and the composition of bondholders. We develop a two-period model in which domestic and foreign investors demand government bonds denominated in local and foreign currencies. Our analysis shows that the government has incentives to manage the currency composition of the debt to influence the composition of the bondholders. In particular, even in an environment where LC debt provides no insurance, the government finds it optimal to issue LC debt to increase the share of domestic debt.

The empirical analysis supports the predictions of the model. Governments in emerging markets predominantly issue debt in local currency, and domestic investors strongly prefer to hold it. An increase in the supply of local currency bonds is positively correlated with higher domestic debt, reinforcing the link between local currency and domestic debt. These findings highlight the role of currency composition as an instrument to manage bondholder composition, in line with the theoretical framework.

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A Proofs

A.1 Lemma 1

Given debt policy $B_1^\star, B_1$, the portfolio $B_D^\star, B_D$ solve the problem of the domestic investors if and only if:

$$0 = \eta_1\Big(Q^\star(\boldsymbol{B}) - \mathbb{E}_e\left[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\frac{\Lambda(\boldsymbol{B}, e_1)}{e_1}\right]\Big) \tag{39}$$

$$0 = \eta_2\Big(Q(\boldsymbol{B}) - \mathbb{E}_e[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)]\Big) \tag{40}$$

$$B_D = 0 \quad if \quad \eta_1 > 0. \tag{41}$$

$$B_D^\star = 0 \quad if \quad \eta_2 > 0. \tag{42}$$

Proof. We begin the proof with the Euler equations of the domestic investors:

$$q\, u'(c_0 - v(n_0)) = \beta\mathbb{E}\left[(1 - d(s))\frac{u'\,(c_1(s) - v(n_1(s)))}{e_1}\right] + \eta_1 \tag{43}$$

$$q^\star u'(c_0 - v(n_0)) = \beta\mathbb{E}\left[(1 - d(s))u'\,(c_1(s) - v(n_1(s)))\right] + \eta_2 \tag{44}$$

Note it is possible to re-write those Euler equations as:

$$0 = \eta_1\left(q\, u'(c_0 - v(n_0)) - \beta\mathbb{E}\left[(1 - d(s))\frac{u'\,(c_1(s) - v(n_1(s)))}{e_1}\right]\right) \tag{45}$$

$$0 = \eta_2\left(q^\star u'(c_0 - v(1 - n_0)) - \beta\mathbb{E}\left[(1 - d(s))u'\,(c_1(s) - v(n_1(s)))\right]\right) \tag{46}$$

$$B_D = 0 \quad if \quad \eta_1 > 0. \tag{47}$$

$$B_D^\star = 0 \quad if \quad \eta_2 > 0. \tag{48}$$

Next we replace the price functions $Q(\boldsymbol{B}), Q^\star(\boldsymbol{B}), \Lambda(\boldsymbol{B}, e_1)$ to get:

$$0 = \eta_1\left(Q(\boldsymbol{B}) - \beta\mathbb{E}\left[(1 - d(s))\frac{\Lambda(\boldsymbol{B}, e_1)}{e_1}\right]\right) \tag{49}$$

$$0 = \eta_2\left(Q^\star(\boldsymbol{B}) - \mathbb{E}\left[(1 - d(s))\Lambda(\boldsymbol{B}, e_1))\right]\right) \tag{50}$$

$$B_D = 0 \quad if \quad \eta_1 > 0. \tag{51}$$

$$B_D^\star = 0 \quad if \quad \eta_2 > 0. \tag{52}$$

Lastly, we take the conditional expectation of ν over each value of e_1 with the expectation of default using F_ϕ.

$\square$

A.2 Proposition 1.

Assume $\text{Cov}(e_1^{-1}, \phi) = 0$. Also, let $B_1^\star > 0$, $B_1 \geq 0$. Then $B_D^\star > 0$ only if $B_D = B_1$.

Proof. We will prove that $B_D = B_1$ given that $B_D^\star > 0$. The proof is by contradiction. Suppose not. Because $B_D < B_1$, foreign investors are marginal investors in both markets. It implies:

$$q = q^\star + \text{Cov}(e_1^{-1}, (1-d)) = \mathbb{E}\left(\frac{(1-d)}{R}\frac{1}{e_1}\right) \tag{53}$$

Then we use $\text{cov}(x, y) = \mathbb{E}(x)\mathbb{E}(y) - \mathbb{E}(xy)$ to re-write the Euler equation for debt in LC of the domestic investors as:

$$(q^\star + \text{Cov}(e_1^{-1}, (1-d)))\, u_0' = \mathbb{E}\Big[u_1'(1-d)\Big] + \text{Cov}(e_1^{-1}, u_1'(1-d)) + \mu_1 \tag{54}$$

$$q^\star u_0' - \mathbb{E}\Big[u_1'(1-d)\Big] = \text{Cov}(e_1^{-1}, u_1'(1-d)) + u_0'\text{Cov}(e_1^{-1}, (1-d)) + \mu_1 \tag{55}$$

where $u_0' = u'(c_0 - v(n_0))$ and $u_1' = u'(c_1 - v(n_1))$. This implies that if domestic investors are not constrained in the FC market, that is $\eta_1 = 0$, then:

$$0 = \text{Cov}(e_1^{-1}, u_1'(1-d)) + u_0'\text{Cov}(e_1^{-1}, (1-d)) \tag{56}$$

This is the optimal condition for the demand for LC debt by domestic investors. Next, we will analyze this optimality condition.
Claim: $\text{Cov}(e_1^{-1}, (1-d)) > 0$.

Proof. The proof is a contradiction. So, assume $\text{Cov}(e_1^{-1}, (1-d)) < 0$. It implies:

$$\frac{\partial \hat{\phi}}{\partial e_1} > 0 \tag{57}$$

So:

$$\frac{\partial \hat{\phi}}{\partial e_1} = \frac{\partial V^R}{\partial e_1} > 0 \tag{58}$$

$$u_1'(1 - v_1')\frac{\partial N^R}{\partial e_1} - u_1' B^F > 0 \tag{59}$$

We know $u_1' B^F > 0$. Then $u_1'(1-v_1')\frac{\partial N^R}{\partial e_1} > 0$. Also, using the first-order condition of labor:

$$1 - v_1' = \tau > 0 \tag{60}$$

Which implies $u_1'(1-v_1') > 0$. So $\frac{\partial N^R}{\partial e_1} > 0$.
From the equilibrium in the labor market (14):

$$\frac{\partial N^R}{\partial e_1} = -\frac{1 - \frac{1}{N^R}}{1 - \frac{\frac{B_1}{e_1} + B_1^\star}{(N^R)^2}} \frac{B_1}{e_1} \tag{61}$$

Note $\frac{\partial N^R}{\partial e_1} < 0$ if $\frac{\frac{B_1}{e_1} + B_1^\star}{(N^R)^2} > 1$ and $N^R < 1$. Which leads to a contradiction.

□

As a result:

$$\text{Cov}(e_1^{-1}, u_1'(1-d)) = -u_0' \text{Cov}(e_1^{-1}, (1-d)) > 0 \tag{62}$$

The last inequality implies $\text{cov}(c_1^R, e_1^{-1}) > 0$, which is a contradiction to (59). This contradiction completes the proof of claim 1.

□

A.3 Lemma 2.

Let $B_1^\star > 0$. Then $\omega(B_1, B_1^\star) > 0$

Proof. First, by Proposition 1, we know $\mu_1 = 0$. So the Euler equations of domestic investors for debt in FC is:

$$Q^\star = \mathbb{E}\big[(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\big] \tag{63}$$

In this price, we use the fact that the non-arbitrage condition of FC debt implies that the SDF of foreign and domestic investors is equally evaluated at the payments distribution of FC debt. Also, using Proposition 1, $\mathcal{B}_F(B_1, B_1^\star) = 0$. Then $\omega(B_1, B_1^\star)$ simplifies to

$$\omega(B_1, B_1^\star) = \frac{\partial Q^\star}{\partial B_F^\star} \mathcal{B}_F^\star(B_1, B_1^\star) \tag{64}$$

Also, from the price function $Q^\star$, we have

$$\frac{\partial Q^\star}{\partial B_F^\star} = \mathbb{E}\left[f_\phi(\hat{\phi}(\boldsymbol{B}, e_1) \frac{\partial V^R}{\partial B_F^\star} \right] < 0 \tag{65}$$

Where the last inequality follows because u is strictly decreasing in debt.

□

A.4 Proposition 2.

Assume $\mathrm{Cov}(e_1^{-1}, \phi) = 0$. Let $B_1^\star > 0$. Then, $B_1 = 0$ cannot be part of the Markov Equilibrium, which completes the proof.

Proof. The proof is by contradiction. Suppose not. Then at $B_1 = 0$ there exist an $B_1^\star$ such that (**??**) and (81) hold.

In this proof, we establish a relationship between the GEE of the government in LC and FC, and we show this relation leads to a contradiction under $B_1 > 0$. In particular, we claim:

$$\begin{aligned} &\frac{\partial n_0}{\partial B}\left[1 - v'(n_0)\right] + \left(\frac{\partial Q}{\partial B} + \frac{\partial Q^\star}{\partial B}\right)\mathcal{B}_F^\star - \mathbb{E}\left((1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\frac{\partial n_1^R}{\partial B}\left[1 - v'(n_1)\right]\right) = \\ &\frac{\partial n_0}{\partial B^\star}\left[1 - v'(n_0)\right] + \left(\frac{\partial Q}{\partial B^\star} + \frac{\partial Q^\star}{\partial B^\star}\right)\mathcal{B}_F^\star - \mathbb{E}\left((1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\frac{\partial n_1^R}{\partial B^\star}\left[1 - v'(n_1)\right]\right) \end{aligned} \tag{66}$$

Proof. To prove this claim, we first use (53):

$$Q = Q^\star + \mathrm{Cov}(e_1^{-1}, (1-d))$$

Also, because $B_L = 0$, then from the problem of the government, we can deduce $\mathrm{cov}(e_1^{-1}, (1-d)) = 0$. Which implies:

$$Q = Q^\star \tag{67}$$

Given that the price of debt in LC and FC are equal, it is direct from (13) that:

$$\frac{\partial n_0}{\partial B} = \frac{\partial n_0}{\partial B^\star} \tag{68}$$

This result implies that the first term in both lines is equivalent. Now, let us focus on the second term. That is, let's analyze the derivatives of the price function. First, note that both prices are equivalent if $B_L = 0$, so their derivative is the same.

$$\begin{aligned} \frac{\partial Q^\star}{\partial B_F} &= \mathbb{E}_e\left[f_\phi(\hat{\phi}(\boldsymbol{B}, e_1)\frac{\partial V^R}{\partial B_F}\right] \\ &= \mathbb{E}_e\left[f_\phi(\hat{\phi}(\boldsymbol{B}, e_1)\frac{\partial V^R}{\partial B_F^\star}\frac{1}{e_1}\right] \\ &= \mathbb{E}_e\left[f_\phi(\hat{\phi}(\boldsymbol{B}, e_1)\frac{\partial V^R}{\partial B_F^\star}\right]\mathbb{E}_e\left[\frac{1}{e_1}\right] \\ &= \mathbb{E}_e\left[f_\phi(\hat{\phi}(\boldsymbol{B}, e_1)\frac{\partial V^R}{\partial B_F^\star}\right] \\ &= \frac{\partial Q^\star}{\partial B_F^\star} \end{aligned} \tag{69}$$

Where the third line follows from the fact that both the default function and the value of repayment are orthogonal to the realization of the nominal exchange rate under $B_1 = 0$. Also, the fourth line follows because the expectation of the nominal exchange rate is one.

Finally, we focus on the last term of the equation. That is, we still need to prove:

$$\mathbb{E}\left((1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\frac{\partial n_1^R}{\partial B}\left[1 - v^{'}(n_1)\right]\right) =$$
$$\mathbb{E}\left((1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\frac{\partial n_1^R}{\partial B^\star}\left[1 - v^{'}(n_1)\right]\right)$$

First, from (14)

$$\frac{\partial n_1}{\partial B} = \frac{\partial n_1}{\partial B^\star}\frac{1}{e_1} \tag{70}$$

Then:

Given 80 and the Euler equations of the government in LC and FC:

$$\omega(B_1, B_1^\star)\frac{\partial \mathcal{B}_F^\star}{\partial B_1} = \omega(B_1, B_1^\star)\frac{\partial \mathcal{B}_F^\star}{\partial B_1^\star} \tag{71}$$

Because $\omega(B_1, B_1^\star) > 0$ we know $\frac{\partial \mathcal{B}_F^\star}{\partial B_1^\star} > 0$. Then:

$$\frac{\partial \mathcal{B}_F^\star}{\partial B_1} > 0 \tag{72}$$

Also, it implies that $\frac{\partial b_1^\star}{\partial B_1} > 0$ where $b_1^\star$ is the solution of debt in FC in the problem of the domestic investors. From the FOC of the domestic investors, we know $b_1^\star$ solves:

$$0 = \left(u_0' - \mathbb{E}u_1'\right)\mathbb{E}\left(1 - d\right) + \mathrm{Cov}(u_1', (1 - d)) \tag{73}$$

We define:

$$m(B_1, b_1^\star) = \left(u_0' - \mathbb{E}u_1'\right)\mathbb{E}\left(1 - d\right) + \mathrm{Cov}(u_1', (1 - d)) \tag{74}$$

Taking the total difference of m and imposing optimality, we have

$$\frac{\partial m}{\partial B_1} + \frac{\partial m}{\partial b_1^\star}\frac{\partial b_1^\star}{\partial B_1} \tag{75}$$

So:

$$\frac{\partial b_1^\star}{\partial B_1} = -\frac{\frac{\partial m}{\partial B_1}}{\frac{\partial m}{\partial b_1^\star}} \tag{76}$$

Also:

$$\frac{\partial m}{\partial B_1} = \frac{\mathbb{E}((1-d)}{\partial B_1}\left(u_0' - \mathbb{E}u_1'\right) + \frac{\left(u_0' - \mathbb{E}u_1'\right)}{\partial B_1}\mathbb{E}\left(\frac{(1-d)}{R}\right) + \frac{\partial \mathrm{Cov}(u_1', (1-d))}{\partial B_1} < 0 \tag{77}$$

And

$$\frac{\partial m}{\partial b_1^\star} = \frac{\mathbb{E}((1-d)}{\partial b_1^\star}\left(u_0' - \mathbb{E}u_1'\right) + \frac{\left(u_0' - \mathbb{E}u_1'\right)}{\partial b_1^\star}\mathbb{E}\left(\frac{(1-d)}{R}\right) + \frac{\partial \mathrm{Cov}(u_1', (1-d))}{\partial b_1^\star} > 0 \tag{78}$$

As a result:

$$\frac{\partial b_1^\star}{\partial B_1} > 0 \tag{79}$$

This leads to a contradiction and completes the proof of the proposition.

□

A.5 Lemma 3

The proof of Lemma 3 is equivalent to proof of Lemma 1 imposing: $B_D^\star > \Phi B_1^\star$

A.6 Proposition 3.

Assume $\text{Cov}(e_1^{-1}, \phi) = 0$. Let $B_1^\star > 0$. Then, $B_1 = 0$ is part of the Markov Equilibrium in a regulated economy.

Proof. From proof of Proposition 2, we know:

$$\frac{\partial n_0}{\partial B}\left[1 - v^{'}(n_0)\right] + \left(\frac{\partial Q}{\partial B} + \frac{\partial Q^\star}{\partial B}\right)\mathcal{B}_F^\star - \mathbb{E}\left((1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\frac{\partial n_1^R}{\partial B}\left[1 - v^{'}(n_1)\right]\right) = \tag{80}$$
$$\frac{\partial n_0}{\partial B^\star}\left[1 - v^{'}(n_0)\right] + \left(\frac{\partial Q}{\partial B^\star} + \frac{\partial Q^\star}{\partial B^\star}\right)\mathcal{B}_F^\star - \mathbb{E}\left((1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\Lambda(\boldsymbol{B}, e_1)\frac{\partial n_1^R}{\partial B^\star}\left[1 - v^{'}(n_1)\right]\right)$$

Then absent wedge ($\omega(B_1, B_1^\star) = 0$), this equivalence implies that if the Euler equation holds at any $B_1^\star$ then the Euler equation in LC also holds. This implies that the pair $B_1^\star = 0$ is a solution to the problem for the government.

□

B Generalized Euler Equation

$$\begin{aligned}
[B_1]: \quad & \left(\frac{\partial N_0}{\partial B_1} + \frac{\partial N_0}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1} + \frac{\partial N_0}{\partial B_D}\frac{\partial B_D}{\partial B_1}\right)\left(1 - \frac{\partial v_0}{\partial B_1}\right) \\
& + \left(\frac{\partial Q}{\partial B_1} + \frac{\partial Q}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1} + \frac{\partial Q}{\partial B_D}\frac{\partial B_D}{\partial B_1}\right)(B_1 - B_D) + Q(\boldsymbol{B}) - \frac{\partial B_D}{\partial B_1}Q(\boldsymbol{B}) \\
& + \left(\frac{\partial Q^\star}{\partial B_1} + \frac{\partial Q^\star}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1} + \frac{\partial Q^\star}{\partial B_D}\frac{\partial B_D}{\partial B_1}\right)(B_1^\star - B_D^\star) - \frac{\partial B_D^\star}{\partial B_1}Q^\star(\boldsymbol{B}) \\
& + \mathbb{E}\left[\left(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\right)\Lambda(\boldsymbol{B}, e_1)\left(\frac{\partial N_1^R}{\partial B_1}\left(1 - \frac{\partial v_1}{\partial B_1}\right) - \frac{1}{e_1} + \frac{1}{e_1}\frac{\partial B_D}{\partial B_1} + \frac{\partial B_D^\star}{\partial B_1}\right)\right] = 0
\end{aligned} \tag{81}$$

$$
\begin{aligned}
[B_1^\star]: \quad & \left(\frac{\partial N_0}{\partial B_1^\star} + \frac{\partial N_0}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1^\star} + \frac{\partial N_0}{\partial B_D}\frac{\partial B_D}{\partial B_1^\star}\right)\left(1 - \frac{\partial v_0}{\partial B_1^\star}\right) \\
& + \left(\frac{\partial Q}{\partial B_1^\star} + \frac{\partial Q}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1^\star} + \frac{\partial Q}{\partial B_D}\frac{\partial B_D}{\partial B_1^\star}\right)(B_1 - B_D) - \frac{\partial B_D}{\partial B_1^\star}Q(\boldsymbol{B}) \\
& + \left(\frac{\partial Q^\star}{\partial B_1^\star} + \frac{\partial Q^\star}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1^\star} + \frac{\partial Q^\star}{\partial B_D}\frac{\partial B_D}{\partial B_1^\star}\right)(B_1^\star - B_D^\star) + Q^\star(\boldsymbol{B}) - \frac{\partial B_D^\star}{\partial B_1^\star}Q^\star(\boldsymbol{B}) \\
& + \mathbb{E}\left[\left(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\right)\Lambda(\boldsymbol{B}, e_1)\left(\frac{\partial N_1^R}{\partial B_1^\star}\left(1 - \frac{\partial v_1}{\partial B_1^\star}\right) + \frac{1}{e_1}\frac{\partial B_D}{\partial B_1^\star} - 1 + \frac{\partial B_D^\star}{\partial B_1^\star}\right)\right] = 0
\end{aligned} \tag{82}
$$

Where we used the definition of the domestic investors' SDF in (7)[7].

Using Lemma 1 or the price function equation, we can replace Q and $Q^\star$ to get:

$$
\begin{aligned}
& \left(\frac{\partial N_0}{\partial B_1} + \frac{\partial N_0}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1} + \frac{\partial N_0}{\partial B_D}\frac{\partial B_D}{\partial B_1}\right)\left(1 - \frac{\partial v_0}{\partial B_1}\right) \\
& + \left(\frac{\partial Q}{\partial B_1} + \frac{\partial Q}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1} + \frac{\partial Q}{\partial B_D}\frac{\partial B_D}{\partial B_1}\right)(B_1 - B_D) + Q(\boldsymbol{B}) - \frac{\partial B_D}{\partial B_1}Q(\boldsymbol{B}) \\
& + \left(\frac{\partial Q^\star}{\partial B_1} + \frac{\partial Q^\star}{\partial B_D^\star}\frac{\partial B_D^\star}{\partial B_1} + \frac{\partial Q^\star}{\partial B_D}\frac{\partial B_D}{\partial B_1}\right)(B_1^\star - B_D^\star) - \frac{\partial B_D^\star}{\partial B_1}Q^\star(\boldsymbol{B}) \\
& + \mathbb{E}\left[\left(1 - F_\phi(\hat{\phi}(\boldsymbol{B}, e_1))\right)\Lambda(\boldsymbol{B}, e_1)\left(\frac{\partial N_1^R}{\partial B_1}\left(1 - \frac{\partial v_1}{\partial B_1}\right)\right)\right] + \left(-Q(\boldsymbol{B}) + Q(\boldsymbol{B})\frac{\partial B_D}{\partial B_1} + Q^\star(\boldsymbol{B})\frac{\partial B_D^\star}{\partial B_1}\right) = 0
\end{aligned} \tag{83}
$$

Next we define the wedges:

$$
\omega(B, B^\star) = \left(\frac{\partial Q}{\partial B_D}\right)(B_1 - B_D) + \left(\frac{\partial Q^\star}{\partial B_D}\right)(B_1^\star - B_D^\star) + \left(\frac{\partial N_0}{\partial B_D}\right)\left(1 - \frac{\partial v_0}{\partial B_1}\right) \tag{84}
$$

$$
\omega^\star(B, B^\star) = \left(\frac{\partial Q}{\partial B_D^\star}\right)(B_1 - B_D) + \left(\frac{\partial Q^\star}{\partial B_D^\star}\right)(B_1^\star - B_D^\star) + \left(\frac{\partial N_0}{\partial B_D^\star}\right)\left(1 - \frac{\partial v_0}{\partial B_1}\right) \tag{85}
$$

So we get:

7. The same derivation steps can be applied to equation (82).

$$
\begin{aligned}
&\left(\frac{\partial N_0}{\partial B_1}\right)\left(1-\frac{\partial v_0}{\partial B_1}\right) \\
&+\left(\frac{\partial Q}{\partial B_1}\right)(B_1-B_D)+Q(\boldsymbol{B})+\omega(B,B^\star)\frac{\partial B_D}{\partial B_1} \\
&+\left(\frac{\partial Q^\star}{\partial B_1}\right)(B_1^\star-B_D^\star)+\omega^\star(B,B^\star)\frac{\partial B_D^\star}{\partial B_1} \\
&+\mathbb{E}\left[\left(1-F_\phi(\hat{\phi}(\boldsymbol{B},e_1))\right)\Lambda(\boldsymbol{B},e_1)\left(\frac{\partial N_1^R}{\partial B_1}\left(1-\frac{\partial v_1}{\partial B_1}\right)\right)\right]=0
\end{aligned} \tag{86}
$$

Re-organizing we have:

$$
\underbrace{\left(\frac{\partial N_0}{\partial B_1}\right)\left(1-\frac{\partial v_0}{\partial B_1}\right)=\mathbb{E}\left[\left(1-F_\phi(\hat{\phi}(\boldsymbol{B},e_1))\right)\Lambda(\boldsymbol{B},e_1)\frac{\partial N_1^R}{\partial B_1}\left(\frac{\partial v_1}{\partial B_1}-1\right)\right]}_{\text{Tax Smooth}}
$$

$$
-\underbrace{\left(\frac{\partial Q}{\partial B_1}\right)(B_1-B_D)-\left(\frac{\partial Q^\star}{\partial B_1}\right)(B_1^\star-B_D^\star)}_{\text{Default Risk}}-\underbrace{\omega(B,B^\star)\frac{\partial B_D}{\partial B_1}-\omega^\star(B,B^\star)\frac{\partial B_D^\star}{\partial B_1}}_{\text{Financial Distortion}}
$$

C Additional Tables and Figures

Table C.1 presents the empirical regularities of sovereign debt in foreign currency for 17 emerging economies. The second and third columns detail each country's share of debt denominated in foreign currency. On average, 26 percent of the debt issued by these countries is in foreign currency. The last two columns detail the foreign investors' portfolio, on average, 54 percent of foreign investors' debt holdings are denominated in foreign currency.

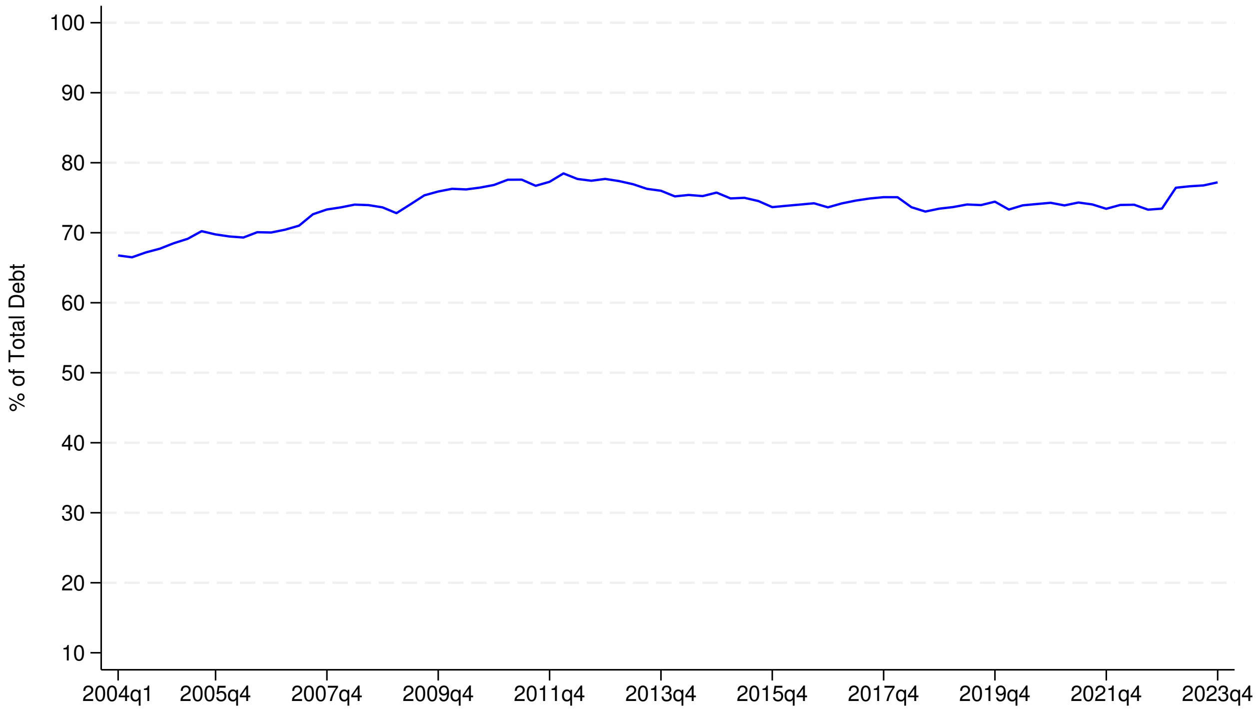
Table C.1: Empirical Regularities of Sovereign Debt by Currency

Country	Total Debt	Share of Debt in FC		Share of Foreign Debt in FC	
	Average (% of GDP)	Average (% of T. Debt)	Δ 2023 - 04 (% of T. Debt)	Average (% of For. Debt)	Δ 2023 - 04 (% of For. Debt)
Argentina	65	60	0	91	1
Brazil	72	7	-24	26	-54
China	45	1	-1	43	-92
Egypt	80	16	27	63	39
Hungary	73	29	5	55	22
India	75	0	0	0	0
Indonesia	32	22	19	51	6
Peru	27	48	-47	66	-36
Philippines	52	26	-13	74	-10
Poland	51	23	-2	53	-1
Romania	31	44	18	76	-9
Russia	15	31	-58	63	-52
South Africa	45	9	-3	29	-31
Thailand	32	1	-7	3	-35
Turkey	37	30	24	64	45
Ukraine	46	51	-14	94	13
Uruguay	56	53	-52	73	-23
Mean	49	26	-8	54	-13
Median	46	26	-2	63	-9
Std. Dev.	19	19	25	27	35

Notes: Average total debt is calculated using total central government debt securities from 2004 till 2023. The share of debt in foreign currency refers to the percentage of total debt securities issued in foreign currency. The share of foreign debt in foreign currency refers to the share of debt securities issued in foreign currency held by foreign investors. The differences are calculated taking the share for 2023q4 and 2004q1, or the first and last observation available for each country.

Figure C.1 shows the average share of total debt denominated in LC for the 17 emerging economies in our sample. During the last 16 years, the debt in LC has averaged between 70 and 80 percent of total debt.

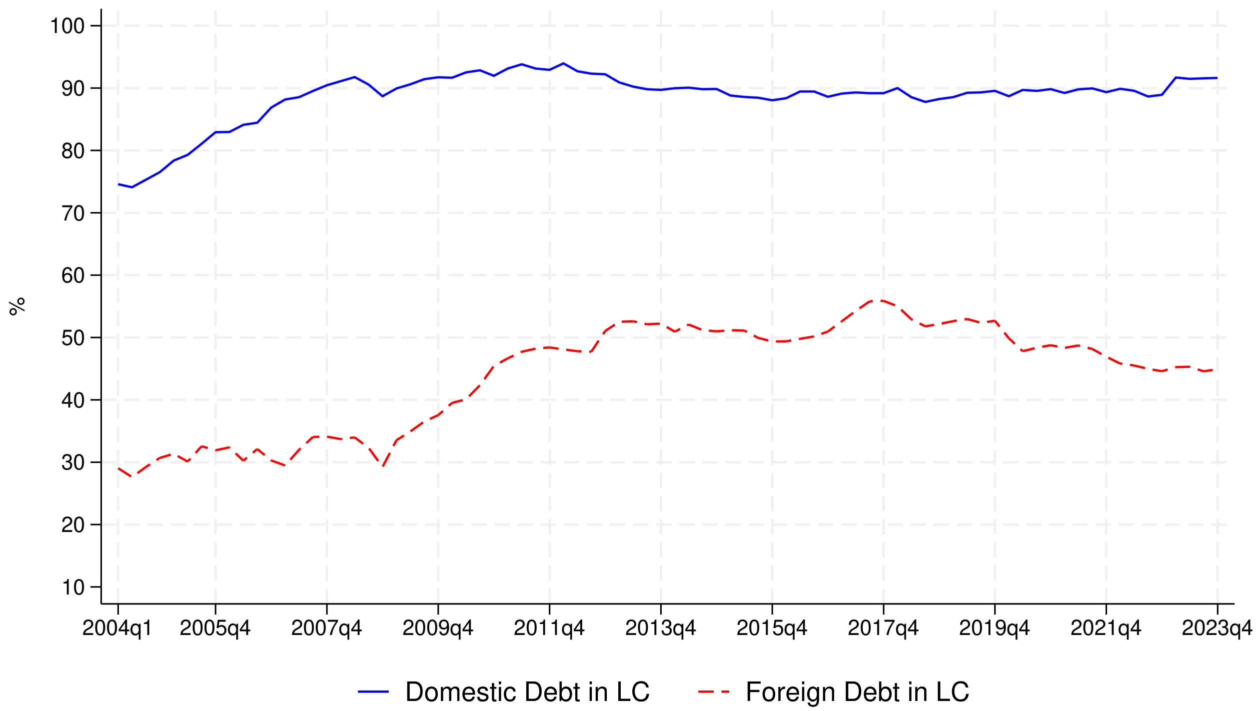
Figure C.1: Share of Total Public Debt in Local Currency



Notes: Arslanalp and Tsuda (2014) and updated on April 30th 2024. Average total debt in LC as a fraction of total debt for the countries in the sample.

In the past 20 years, both domestic and foreign investors have increased their share of debt denominated in LC. Domestic investors' share rose notably in the first few years, while foreign investors increased their LC share after the global financial crisis, after which it stabilized. This pattern is illustrated in Figure C.2.

Figure C.2: Share of Domestic and Foreign Debt in Local Currency



Notes: Arslanalp and Tsuda (2014) and updated on April 30th 2024. Domestic (Foreign) debt in LC is the average domestic (foreign) debt in LC as a fraction of total domestic (foreign) debt for the countries in the sample.